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rubbing of the entire Cross, giving the elaborate patterns on both sides, and the curious inscription along its edge, which he translated; and he then concluded by expressing his conviction, that notwithstanding that during the five days he remained at Iona he was unceasingly occupied, he had, nevertheless, left much unnoticed, with regard to the numerous group of buildings in the vicinity of the cathedral, as well as of many interesting localities in the interior of the island, connected with the history of the celebrated St. Columbkille and the Culdees.

May 12, 1845.

CAPTAIN LARCOM, R. E., Vice-President, in the Chair.

Charles Bournes, Esq., W. C. Dobbs, Esq., Wm. Henn, Esq., D. B. Starkey, Esq., and Benjamin Wilme, Esq., were elected Members of the Academy.

Mr. William Roberts, F. T. C. D., read a paper on some Geometrical Theorems relative to Elliptic Functions.

It is well known that the arc of a spherical ellipse is expressed by an elliptic function of the third order, with a circular parameter: and it has been also proved (see M. Liouville's Journal of Pure and Applied Mathematics, vol. ix.) that the arc of the intersection of a sphere with a cone of the second degree, one of whose external principal axes is a diameter, and whose vertex is situated on the surface, may be made to supply a general representation of the three elliptic transcendants; at least with the aid of the *modular* transformations, which have been given by Lagrange and M. Jacobi. The object of the present communication is to shew that these results are but particular cases of the general theorem, that the arc of any symmetrical intersection of a sphere with a cone of the second order (i. e. when the centre of the sphere lies upon one of the principal axes of the cone) may be expressed by elliptic functions. This is easily deduced from the following formulæ for the rectification of the class of spheric curves represented by the polar equation,

$$\sin^2\rho\,\mathrm{F}(\omega) + \sin 2\lambda \cos\rho = 1 \qquad \text{(a)}$$

where λ is a constant. Let s_1, s_2, denote the two arcs which correspond to the same value of the polar angle ω, and we will have

$$s_1 + s_2 = \cos\lambda \int \sqrt{\left\{\frac{4\,\mathrm{F}^2 + \mathrm{F}'^2 - 4\,\mathrm{F} + \sin^2 2\lambda}{\mathrm{F} - \cos^2\lambda}\right\}}\,\frac{d\omega}{\mathrm{F}}$$

$$s_1 - s_2 = \sin\lambda \int \sqrt{\left\{\frac{4\,\mathrm{F}^2 + \mathrm{F}'^2 - 4\,\mathrm{F} + \sin^2 2\lambda}{\mathrm{F} - \sin^2\lambda}\right\}}\,\frac{d\omega}{\mathrm{F}}:$$

F denoting the derived function of $\mathrm{F}(\omega)$.

Now in the case of the intersection of a cone and sphere, such as we have described, $\mathrm{F}(\omega)$ is a linear function of $\cos 2\omega$, and each of the foregoing integrals is reducible to an elliptic function of the third kind.

In the case of the spherical ellipse, the two arcs s_1 s_2 are equal; and when the vertex of the cone is upon the surface, the second arc s_2 vanishes altogether.

The same relation between the principal angles of the cone, and the distance of its vertex from the centre of the sphere, will cause the parameters in both the functions Π to vanish. And it is remarkable that, in this case, when the sum and difference of the arcs s_1, s_2, are each expressible by a transcendant of the first order, the curve will coincide with the locus of the vertex of a spherical triangle, the base of which is given, and of which the product of the sines of the semi-sides is constant, and less than the square of the sine of the fourth part of the base.

This result is strikingly analogous to M. Serret's theorem on the rectification of the Cassinian curve, published in M. Liouville's Journal, vol. viii. p. 145.

If the moduli of the two functions which express the values of $s_1 \pm s_2$ be complementary, the curve will coincide with the locus of the vertex of a spherical triangle whose base is given, and of which the product of the tangents of the semi-sides is constant, and less than the square of the tangent of the fourth part of the base.

Equation (a) may be transformed into

$$\tan^4 \tfrac{1}{2}\rho - 2\tan^2 \tfrac{1}{2}\rho \, \mathrm{F}(\omega) + \frac{1-\sin 2\lambda}{1+\sin 2\lambda} = 0,$$

where $\mathrm{F}(\omega)$ simply is written for $\frac{1-2\mathrm{F}(\omega)}{1+\sin 2\lambda}$. The class of plane curves whose polar equation

$$r^4 - 2r^2 \mathrm{F}(\omega) + c^4 = 0$$

is analogous to the above, may be rectified by similar formulas. These are

$$s_1 + s_2 = \frac{1}{\sqrt{2}} \int \sqrt{\left(\frac{4\mathrm{F}^2 + \mathrm{F}'^2 - 4c^4}{\mathrm{F} - c^2}\right)} \, d\omega$$

$$s_1 - s_2 = \frac{1}{\sqrt{2}} \int \sqrt{\left(\frac{4\mathrm{F}^2 + \mathrm{F}'^2 - 4c^4}{\mathrm{F} + c^2}\right)} \, d\omega.$$

Each of these integrals will be reducible to an elliptic function of the third order, provided that $\mathrm{F}(\omega)$ be a linear function of $\cos 2\omega$. The following curves, among others, possess this property: first, the locus of a point, such that, tangents being drawn from it to two equal non-intersecting circles, their rectangle is constant, and less than the square of the tangent drawn to either from the point midway between their centres: secondly, the locus of the intersection of tangents to an ellipse which include a given angle. This curve is composed of two closed branches concentric with the ellipse, and satisfying the given condition by angles which are supplemental.

The well-known property of the lemniscate may be extended to the locus of the orthogonal projections of the centre of an ellipse or hyperbola in general upon its tangents. First, in the case of the ellipse, the curve may be derived by taking the locus of a point, such that tangents being drawn from it to two equal intersecting circles, their rectangle may be constant, and equal to the square of half the common chord.

If the circles do not intersect, and if the rectangle under the tangents be equal to the square of the tangent to either from the middle point between the centres, the locus will give the curve derived from an hyperbola, whose real axis is greater than the imaginary.

To get the curve when the imaginary axis is greater than the real, we must take the locus of a point, such that lines being drawn from it to meet the circumferences of two equal, non-intersecting circles, and subtending right angles at their centres, their rectangle may be constant, and equal to the sum of the squares of the radius, and of half the distance between the centres.

There exist in spherical geometry numerous properties of an analogous character.

The Rev. Charles Graves made a communication relative to the new functions employed by him in the interpretation of his theory of Algebraic Triplets.

The three functions A, M, N, defined in p. 60, possess so many interesting properties that they seem to deserve distinctive appellations. The first is symmetrical with respect to its two amplitudes, ϕ and χ; and its properties are in the main analogous to those of the trigonometrical function *cosine.* On the other hand, M and N are not symmetrical functions of ϕ and χ; but either of them may be obtained from the other by interchanging the two amplitudes; and they correspond in many respects to the trigonometrical *sine.* Mr. Graves proposes then to call A the *cotresine,* and M and N the *tresines* of ϕ and χ: and he designates them respectively by the symbols cotr $[\phi, \chi]$, tres $[\phi, \chi]$, tres $[\chi, \phi]$.

As in trigonometry $\cos\phi = \cos(2i\pi + \phi)$, and $\sin\phi =$